Mathematical Techniques A: Solutions Exercise Set 3

November 5, 2015

Exercise 3

Higher order derivatives and identifying maxima and minima of a function

1. By differentiation find the turning point of the function $y = 2x^2 + 12x + 1$, and determine whether it is a maximum or a minimum. Sketch the graph of the function.

2. By differentiation find the turning point of the function $y = x^2 + x - 6$, and determine whether it is a maximum or a minimum. Sketch the graph of the function.

3. Show by differentiation that the graphs of $y = x^2 + 6x + 9$ and $y = -x^2 - 6x - 9$ are both tangent to the x - axis at $x = -3$, but that the first of these functions reaches a minimum at this point, and the second a maximum. Confirm your result by solving the equations $x^2 + 6x + 9 = 0$ and $-x^2 - 6x - 9 = 0$. Sketch the graphs of the functions.

4. Show by differentiation that the function $y = x^3 + 2x^2 + x + 9$ has a maximum at $x = -1$, a minimum at $x = -1/3$, and a point of inflexion at $x = -2/3$. Sketch the function.

5. By examining the signs of the first and second derivatives, determine the slope and curvature of the function $y = -x^3 + 9x$ at the point $x = 4$. Hence draw a rough sketch graph of the curve in the vicinity of this point.

6. By examining the signs of the first and second derivatives, determine the slope and curvature of the function $y = x^2 + 5x + 3$ for values of x lying between 0 and 1. (A range of values such as this is called an interval). Hence draw a rough sketch graph of the curve in the interval $0 < x < 1$.

7. Given the function $y = 1/x$, examine the signs of the first and second derivatives and hence show

(i) that the function is negatively sloped for all $x \neq 0$;

(ii) that the function is convex when $x > 0$ and concave when $x < 0$.

(iii) Use this information to sketch the graph of the function.