1 Quantitative Techniques for Economics

Homework 3

This homework is due on **Wednesday 26 October before 10am.** You have two options: 1) Leave the homework in my mailbox (MF402). 2) Send me an email with the solutions. I will not accept only codes (matlab files or similar) as an answer. You will have to write or type the results. Code attached at the end of the homework will be appreciated (again, do not send me codes by email). You can work on it in groups and turn in a single copy for each group. Answer clearly.

Question 1.

Consider the following economy. The setting is the neoclassical growth model. The problem is to maximize

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to

$$c_t + k_{t+1} = e^{z_t} k_t^{\alpha} + (1-\delta) k_t.$$

where k_0 and z_0 are given. The technology level z_t follows the following process:

$$z_t = \rho_z z_{t-1} + \sigma \varepsilon_t$$

Assume the following parameter values:

$$\begin{aligned} \beta &= 0.99 \\ \alpha &= 0.33 \\ \delta &= 0.06. \end{aligned}$$

1) Write the problem in recursive form. What do we know about the value function of this problem? Find the deterministic steady state value of capital, k_{ss}, consumption, c_{ss}, and of the life-time utility function, V_{ss}.

2) **Value function iteration with a Fixed Grid:**
Assume that the process for technology z_t is discretized by the following two-state Markov process. :

$$z = \begin{cases} -0.03 & \text{if } s_t = 1 \\ 0.03 \text{ if } s_t = 0 \end{cases},$$

where s_t indicates the two states of the world, and the transition probability matrix is:

$$\Pi = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}.$$

Fix a grid of 500 points capital centered around the deterministic steady state value of capital, k_{ss}, with a coverage of $\pm 25\%$ of k_{ss} and equally spaced. Iterate on the value function implied by the social planner's problem until the change in the sup norm between the iterations is less than 10^{-5}. Use V_{ss} as an initial guess. Compute the policy function for next-period capital and consumption. How many iterations do you need for convergence? How many seconds? Plot in the same graph the policy functions of consumption (y-axex) with respect to capital (x-axes) for each of the two exogenous states of the world.

3) **Value function iteration with a Fixed Grid: more states**

Assume now that the process for technology z_t is discretized by the following five-state Markov process:

$$z = \begin{cases} -0.0673 & \text{if } s_t = 1 \\ -0.0336 & \text{if } s_t = 2 \\ 0 & \text{if } s_t = 3 \\ 0.0336 & \text{if } s_t = 4 \\ 0.0673 & \text{if } s_t = 5 \end{cases},$$

where s_t indicates the five states of the world, and the transition probability matrix is:

$$\Pi = \begin{bmatrix} 0.9727 & 0.0273 & 0 & 0 & 0 \\ 0.0041 & 0.9806 & 0.0153 & 0 & 0 \\ 0 & 0.0082 & 0.9863 & 0.0082 & 0 \\ 0 & 0 & 0.0153 & 0.9806 & 0.0041 \\ 0 & 0 & 0 & 0.0273 & 0.9727 \end{bmatrix}.$$

As in the previous question, fix a grid of 500 points for capital centered around the deterministic steady state value of capital, k_{ss}, with a coverage of $\pm 25\%$ of k_{ss} and equally spaced. Iterate on the value function implied by the social planner's problem until the change in the sup norm between the iterations is less than 10^{-5}. Use V_{ss} as an initial guess. Compute the policy function for next-period capital and consumption. How many iterations do you need for convergence? How many seconds? Plot in the same graph the policy functions of consumption (y-axex) with respect to capital (x-axes) for each of the five exogenous states of the world.

4) **Accelerator.**

Recompute your solution of question (3), i.e. with the 5 state Markov-chain and the 500 grid-points for capital, using an accelerator, i.e. skipping the max operator in the Bellman equation 19 out of each 20 times. How many iterations do you need for convergence? How many seconds? Comment your findings and compare them with the answer in part (3).

5) **Multigrid**:

Considering the same setting as in question (3), implement a multigrid scheme with grids centered around k_{ss} with coverage of $\pm 25\%$ of k_{ss} and equally spaced. Follow these three steps:

a) Compute the first grid of 50 points. Iterate on the value function implied by the social planner's problem until the change in the sup norm between the iterations is less than 10^{-3} (Notice that we require a much looser converge criterion for the first grid). Use V_{ss} as an initial guess. Call V^1 the converged value function.

b) Compute the second grid of 200 points. Use a linear interpolation of V^1 to derive an initial guess V_0 (In other words: you have the converged matrix V^1 50x5. Find a matrix 200x5 whose values are obtained by linearly interpolating the values of V^1).Use the obtained matrix as an initial guess for the value function iteration. Iterate on the value function implied by the social planner's problem until the change in the sup norm between the iterations is less than 10^{-3} (Notice that we require a much looser converge criterion also for the second grid). Call V^2 the converged value function.

c) Compute the third grid of 500 points. Use a linear interpolation of $V^2$to derive an initial guess V_0 (In other words: you have converged matrix V^2 200x5. Find a matrix 500x5 whose values are obtained by linearly interpolating the values of V^2).Use the obtained matrix as an initial guess for the value function iteration. Iterate on the value function implied by the social planner's problem until the change in the sup norm between the iterations is less than 10^{-5} (Notice that now we required the same converge criterion as in question (3)).

How many iterations in the last step do you need for convergence? How many seconds does the whole-procedure require? Comment your findings and compare them with the answer in part (3).