Introduction To Quantitative Economics: Problem Set 5

December 11, 2015

Question 1

While grading a final exam a professor discovers that two students have virtually identical answers. He talks to each student separately and tells them that he is sure that they shared their answers, but he cannot be sure who copied from whom. He offers each student a deal if they both sign a statement admitting to the cheating, each will be given an F on the course. If only one signs the statement, he will be allowed to withdraw from the course and the other non-signing student will be expelled from the university. Finally, if neither signs the statement they will both get a C for the course because the professor does not have enough evidence to prove that cheating has occurred. Assuming that students are not allowed to communicate with each another, set up the relevant payoff matrix. Does each student have a dominant strategy?

Question 2

A competitive firm has the cost structure described in the following table. Graph the MC, AVC and ATC curves. How many units of output will it produce at a market price of 32? Calculate its profits and show them in your graph.

Q	ATC	AVC	MC
1	44	4	8
2	28	8	16
4	26	16	32
6	31	24	48
8	37	32	64

Question 3

If the short run marginal and average variable cost curves for a competitive firm are given by $SMC = 2 + 4Q$ and $AVC = 2 + 2Q$, how many units of output will it produce at a market price of 0? At what level of fixed cost will this firm earn zero economic profit?

Question 4

Bertrand duopolists face a market demand curve given by P = 56-2Q. Each can produce output at a constant marginal cost of 20/unit. Find the equilibrium price and quantity.

Question 5

The market demand curve for a pair of Bertrand duopolists is given as $P = 15 - Q$, where $Q = Q1 + Q2$. The constant per unit marginal cost is 18 for each duopolist. Find the equilibrium price, quantity and profit if there is Bertrand price competition between the two duopolists.

Question 6

Firm 1 and firm 2 are automobile producers. Each has the option of producing either a full size SUV or a mid size SUV. The payoffs to each of the four possible combinations of choices are as given in the following payoff matrix. All decisions are made simultaneously (i.e. without knowing what the other firms choice).

Payoff Matrix

		Firm 2	
		Full Size	Mid Size
Firm 1	Full Size	800 800	1600 2000
	Mid Size	2000 1600	1000 1000

1. Does either firm have a dominant strategy?
2. What are the Nash equilibria to this game. Describe your reasoning.

Solutions

Question 1

In the table below we can observe the pay-offs contingent to each of the strategies.

Payoff Matrix

		Student 2	
		Not Sign	Sign
Student 1	Not Sign	C C	Withdraw Expelled
	Sign	Expelled Withdraw	F F

To determine if there is a dominant ranking we need to assume a ranking between alternative.

- withdraw$> C > F >$expelled, for both students
- $C >$withdraw$> F >$expelled, for both students
- $C >$withdraw$> F >$expelled for one of them and withdraw$> C > F >$expelled for the other

A dominant strategy exist only when withdraw$> C$. Thus in case one both of them have a dominant strategy to sign. In the second case, we no dominant strategy. For the third one only the one with preference for withdraw$> C$ has a dominant strategy to sign.

Question 2

In Figure 1 we see the plots of ATC, AVC and marginal costs. The profit maximizing is where $MC = MR = P$. This point corresponds to $Q = 4$. The yellow shaded area in the plot is representing the profits as it shows the average mark up per unit times the units:

$$\Pi = TR - TC = 4(32 - 26) = 24$$

Question 3

Since profit is total revenue minus total cost:

$$\Pi = Q(AR - AVC) = Q(10 - 2 - 2Q) = 8Q - 2Q^2$$

Maximizing the profits we have:

$$\max_Q \Pi \Rightarrow 8 - 4Q = 0 \Rightarrow Q = 2$$

the profits in this case are $\Pi = 16 - 8 - FC = 8 - FC$ thus with $FC = 8$ the economic profit will be zero.

Question 4

Given that they both sell the same product and the MC are constant which means $MC = AC$, they will both a price equal to their $MC = P = 20$. The reasoning is as follows:

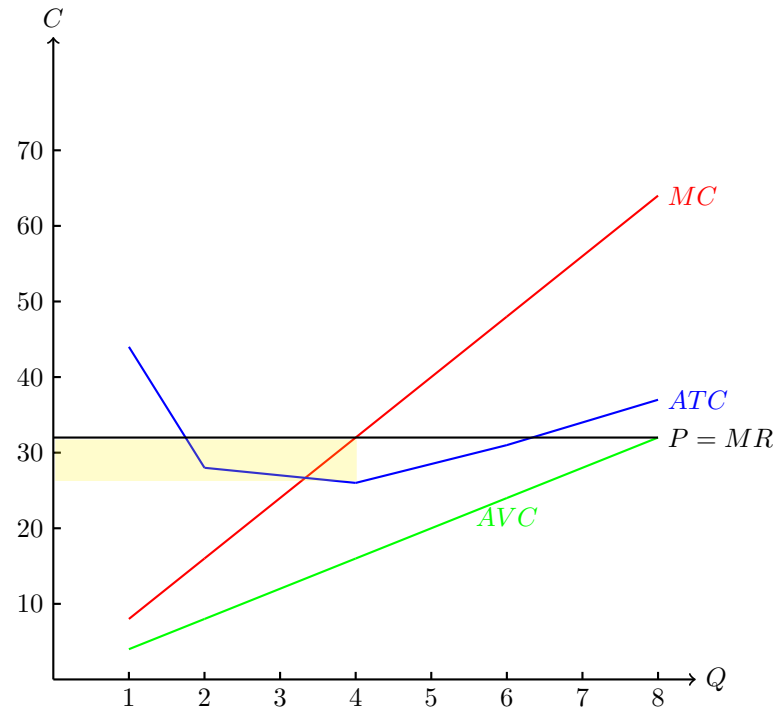

Figure 1: Plot of ATC, AVC, MC and Profits.

- They both have no incentive to charge $P < AC$ as they will be making a loss
- If one of them charges a higher price $P > AC$, the other one has an incentive to undercut the price by ϵ and get all the consumers and profits.
- Thus for both of them the optimal strategy is to charge $P = AC = MC$

Given the price of 20 we can find that $20 = 56 - 2Q \Rightarrow Q = \frac{20-56}{-2} = 18$ and assuming that they share the market equally we have $Q_1 = Q_2 = 9$.

Question 5

For the same reasoning as in question 4, we have that $P = AC = 18$. Then $Q = -3$ which means that at the price of 18 there will be no demand. Thus the quantity sold will be zero. As neither the firms are willing to produce at a loss and the consumers are not willing to buy at the price of 18.

Question 6

Payoff Matrix

		Firm 2: Full Size	Firm 2: Mid Size
Firm 1	Full Size	800, 800	1600, 2000
Firm 1	Mid Size	2000, 1600	1000, 1000

1. No player has a dominant strategy. The row player would prefer row (1) if he assumes that the column player would play column (2) (as $2000 > 1000$) and row (2) if he assumes that the column player would play column (1) (as $1600 > 800$). Similarly for the column player.

2. The two Nash equilibria are $(1600, 2000)$ and $(2000, 1600)$. To see this, observe that the pay-offs from deviations from the Nash equilibria are always lower than the Nash pay-off given the strategy of the other player (as $800 < 1600$ and $1000 < 2000$)