

## EC108 - Problem set for Professor Moav's section

### Question 1

1. Consider the following production function:

$$Y = F(K, L) = AK^{1/2}L^{1/2}$$

where  $Y$  is total output,  $K$  is capital and  $L$  is labour.

Find output per worker as a function of capital per worker:  $y = f(k)$ , where  $y = Y/L$  is output per worker,  $k = K/L$  is capital per worker, and  $f$  is the production function of output per worker as a function of capital per worker.

2. The marginal product of capital is defined as the rate at which output increases when capital increases. In other (less accurate) words, it is the change in output resulting from a small change in capital, calculated per unit of capital. This rate depends on the quantity of capital and labour. Find the marginal product of capital. To calculate the marginal product of capital, take the derivative of  $y$  with respect to  $k$ . i.e.,  $f'(k)$ .

3. Analyse the Solow model of country  $a$ : assume that the parameters in country  $a$  are as follows: the rate of depreciation  $\delta = 5\%$ , the productivity coefficient  $A = 1$ , and the saving rate is  $s = 10\%$ . Suppose that in the initial period, period one,  $k_1 = 2$ .

Use a Spreadsheet Software Program, such as Excel, to simulate the evolution of the economy.

Instructions: in the first column, column A, write the period: in A1 write "period" in A2 insert the number 1, in A3 insert "=1+A2". In column B calculate  $k$ . it will be 2 in B2 (just under the title  $k$ ). B3 will be the calculated  $k$  in period 2. In column C calculate output per worker. In case you didn't find the correct answer to part one above it is simply  $k^{1/2}$ . In column D calculate consumption per worker. It is income per worker  $c$  (equal to output per worker) net of savings.  $c = (1 - s)y$ . In the next column calculate net investment per worker. It is investment per worker net of depreciation per worker. It is thus output per worker multiplied by the saving rate minus depreciation. Depreciation is simply the depreciation rate  $\delta$  multiplied by  $k$ , so net investment is  $sy - \delta k$ . Now you can calculate  $k$  in the second period. It is  $k$  from the previous period + net investment. Copy the first row of data (row number 2) excluding the first two columns to the next row. Now you can copy the entire second row of data (row 3) to the next 200 rows.

Plot a figure of output, capital, investment and consumption over time (10 periods are sufficient).

4. Calculate the growth rate of output over time. The growth rate of a variable is the change in the variable divided by the variable, and in particular, in our case:  $(y_{t+1} - y_t)/y_t$ . (You can add a column to the spreadsheet with the growth rate calculation).

5. What is the level of capital per worker, output per worker, and consumption per worker after 200 periods?
6. Calculate the level of capital per worker, output per worker, and consumption per worker in the steady state.
7. Country  $b$  is identical to country  $a$  in all parameters excluding the saving rate, which is 20%.
  - a. Find the growth path of output per worker in country  $b$  as you did for country  $a$ .
  - b. Plot in a figure, over 10 periods, output per worker in the two countries.
  - c. Plot in a figure, over 10 periods, consumption per worker in the two countries.
  - d. Compare the steady state output and consumption of the two countries.

### Question 2

1. Plot a figure, based on the parameters of “country  $a$ ” from Question 1, of output per worker, investment per worker and depreciation per worker as functions of capital per worker. (i.e.,  $kt$  is on the horizontal axis).
2. Add to the figure the steady state level of capital. Show on the figure consumption in the steady state.
3. Use the figure to show the effect of an increase of the saving rate from 10% to 20% on the steady state level of capital and consumption.
4. What is your opinion on a policy that encourages an increase of the saving rate from 10% to 20%? (Consider in your answer the long run benefit and the short run cost).
5. Does an increase in the saving rate always lead to higher consumption in the steady state?
6. Use the figure to show the effect of an increase in the depreciation rate on the steady state.
7. What will happen to the steady state level of capital and output if both the saving rate and the depreciation rate double? (saving rate increases from 10% to 20%, and the depreciation rate increases from 5% to 10%). What happens to welfare?

### Question 3

Consider an economy in steady state according to the Solow model. As a result of a change in policy, there is an instant increase in the size of the working population due to immigration.

1. Plot a figure with  $k$  on the horizontal axis and output per worker, investment per worker and depreciation per worker on the vertical axis. Show the steady state of the economy before the immigration. Show the change in the economy on the figure after the immigrants have joined the labour force. What will happen in the long run?

2. Plot a figure of output per worker  $y$  over time. Start before the immigration event, and continue until the economy reaches the new steady state.
3. Repeat (2) above for total output  $Y$ .

**Question 4**

The Malthusian Model

Consider an economy in which output is a function of labour  $L$  and land  $K$ , which is constant:  $Y = AK^\alpha L^{1-\alpha}$

The productivity coefficient  $A$  is increasing by 10% each period (think of a period as a generation). According to the Malthusian model (population size adjusts such that income is at subsistence):

- a. Income per capita will grow at a rate of 10% each period
- b. Population growth will be 10% each period
- c. Total output will grow at a rate of more than 10% each period
- d. Income per capita will decline over time

One of the statements a, b, c or d is correct. Which one is it? Explain.